Structural Optimization of Laminated Conical Shells

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A methodology for determining an optimal design of a laminated structure is described in terms of its application to the design of an upper-stage solid rocket exit cone. Structural design is complicated by the conical geometry of the exit cone and the associated available fiber wrapping techniques. The methodology described is comprised of three independent components: 1) the field theory, 2) a statistical sampling scheme for constructing surrogate structural response functions, and 3) a dynamic programming model for accomplishing the optimization. Use of the first and implementation of the second and third of these components is described. A design solution that provides a 37 lb weight savings over the base line aluminum design is identified.

Introduction

THE spectrum of applications for composite materials is growing rapidly because such materials can be tailored to meet design performance specifications at a reduced weight. Successful implementation of a composite material in a particular application often hinges upon the availability of a method for optimal structural design. A methodology for determining a near-optimal structural design of laminated components is described here. The method is comprised of three independent components: 1) the basic field theory as embodied in a finite element or finite difference code, 2) a statistical sampling scheme for estimating structural performance of any design, and 3) a dynamic programming optimization model for identifying an optimal design. While the method and each of its components are general, they are developed here in terms of their application to an advanced upper-stage rocket motor exit cone.

The design of a composite structure involves the selection of a reinforcement architecture or lamination pattern that will resist both the direction and magnitude of applied loads while conforming to a specific geometric envelope. Previous methods of laminate optimization address the design of trusses, flat panels, or cylindrical shells.[1,2] The shape of an exit cone makes the selection of a lamination pattern more difficult. The conical geometry dictates the reinforcement trace in which the fiber orientation relative to the cone axis changes as the fiber is wound. In addition, by virtue of the task for which it is used, an exit cone is subject to a variety of failure modes that change over time and among which the governing mode is not known a priori. Consequently, previously developed methods for laminate design cannot be used to select an optimal exit cone design.

An extensive literature on optimization in general and structural optimization in particular exists. Venkayya[3] provides a survey of structural optimization methods from which it is apparent that the available techniques are usually based upon a sequence of functional evaluations or approximations and upon a rule for selecting subsequent function evaluations. The disadvantage of this approach is that the number of function evaluations can be so large that the computational effort required to identify an optimal design can be prohibitive. In the methodology described here, a statistical sampling scheme is used to construct a surrogate structural performance function. The use of the sampling plan fixes the number of evaluations of the true performance function, thereby assuring that the required computational effort is economically acceptable. The surrogate function is then subjected to optimization.

Once the surrogate performance function is defined, any of several optimization methods may be used to determine an optimal design. In the case of the exit cone under study, there is a natural partitioning of the cone into sectors on the basis of the dominant load mechanisms. Consequently, the resulting optimization problem is highly conducive to the use of dynamic programming. Therefore, a dynamic program is used to obtain a design that is optimal for the entire structure by synthesis of compatible sector-by-sector lamination patterns.

Problem Definition and Field Theory

The problem is to identify a weight-efficient design for a laminated conical shell that will meet or exceed the performance requirements for a base line solid rocket exit cone. The base line structure is illustrated in Fig. 1. The geometric envelope of the structure is defined by motor ballistic requirements. The key dimensions of throat diameter, half-angle, and exit plane diameter are specified. The design objective is to tailor the stiffness and thermal expansion characteristics of the composite to achieve optimum strength.

Examination of the mission performance criteria of the exit cone indicates that the set of dominant loading mechanisms changes along the length of the cone. Consequently, the exit cone is partitioned into three sectors within each of which the significant loads are the same. The three sectors, designated forward, middle, and aft, are illustrated in Fig. 2. Also indicated in the figure are some of the important loading conditions for each sector. One of the significant differences among the three sectors is that the primary load in the forward sector is thermal, while in the midsector it is an asymmetrically applied axial load and in the aft sector a circularly symmetric bending one.

The variables that describe a particular laminated composite architecture are the reinforcing fiber properties, matrix properties, lamina fiber orientations, lamina thicknesses, lamina

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stacking sequences, and the number of laminae. As illustrated in Fig. 3, several simplifying assumptions define a set of preselected laminate variables and thereby identify the design variables to be determined. The reinforcement fiber is assumed to be Hercules Incorporated HM graphite. Eight 20 mil (0.5 cm) layers are assumed so that total shell thickness is 160 mil (~ 4 cm). This dimension conforms to the base line design. Each layer is made up of two plies that are oriented at angles of $\pm\theta$. In order to conform to the constitutive requirements of the finite element and finite difference codes, stacking sequences are restricted to designs that are symmetric about the center plane of the shell. Within these assumptions, the design variables are the angle ply orientationss, the selection of which implicitly defines the stacking sequence. For the purpose of the present analysis, values of the angle of ply orientation are restricted to 15 deg increments so that each variable must take one of seven values (0-90 deg in 15 deg increments).

Within the framework of the stated problem assumptions, the design optimization problem is to select values of the design variables

$$\theta_{ij} \quad i=1,2,3; \quad j=1,2,\ldots,8$$

the angle of ply orientation in layer j of sector i. The assumption of symmetry about the center plane reduces the number of variables by one-half so that the final number of design variables is 12. The measure of performance for any selected design is the minimum factor of safety over all active loads.

For the defined optimization problem, there are $7^4 = 2401$ solutions in each sector. For three sectors, the total number of feasible lamination patterns is 2401^3. However, available fabrication techniques restrict the choice of combinations of orientation angles. Existing methods of fiber winding dictate the achievable fiber traces from sector to sector and thereby constrain the set of feasible designs. As illustrated in Fig. 4, five fabrication methods are assumed. Restriction of the selection of ply orientation angles to those consistent with the fabrication constraint reduces the total number of feasible solutions to 14^4. While application of the fabrication constraint dramatically reduces the number of feasible solutions, there are still far too many candidate solutions to permit exhaustive enumeration.

Classical laminate theory[4] provides a convenient method for manipulating the stiffness and thermal strain tensors. As illustrated in Fig. 5, a thin-shell approximation to the constituitive laws is assumed with the result that the 6×6 stiffness matrix is reduced to a 3×3 matrix. The reduced Q matrix consists of four constants for an orthotropic or balance ply lamina. The corresponding laminate response to the applied loads is illustrated in Fig. 6. The stress-strain field analysis is performed using the BOSOR[5] finite difference code for the middle and aft sectors and, because of the importance of thermal expansion effects, using the SAAS III finite element code for the forward sector. Both codes are limited by the material constituitive laws they employ and their shortcomings are partially corrected by postprocessing of the output strains. In general, for the methodology developed here, any applicable field analysis procedure may be used.

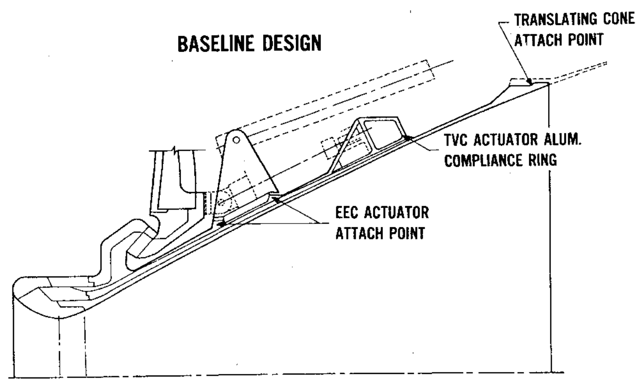


Fig. 1 Base line solid rocket exit cone (attachments indicated by dotted outline).

Design Performance Estimation

Any laminate optimization method will require evaluation of structural response for at least some of the candidate designs. In the absence of a convenient to use closed-form definition of shell response, an efficient approach to design evaluation is required. For any particular laminate, computation of the structural response is accomplished using the above-described elasticity-based field theory. However, these analyses cannot practically be performed for each of the 14^4 candidate designs.

Evaluation of all candidate designs would yield an exact design response surface. In order to construct an estimate of this response surface, an efficient method for judiciously sampling the true surface is employed. A balanced fractional factorial experimental design[7] is used to select a subset of the candidate design cases to be evaluated using the field analysis codes. Each of the selected cases provides one data point to be used in estimating the design response function.

For each sector of the cone, a balanced set of four different

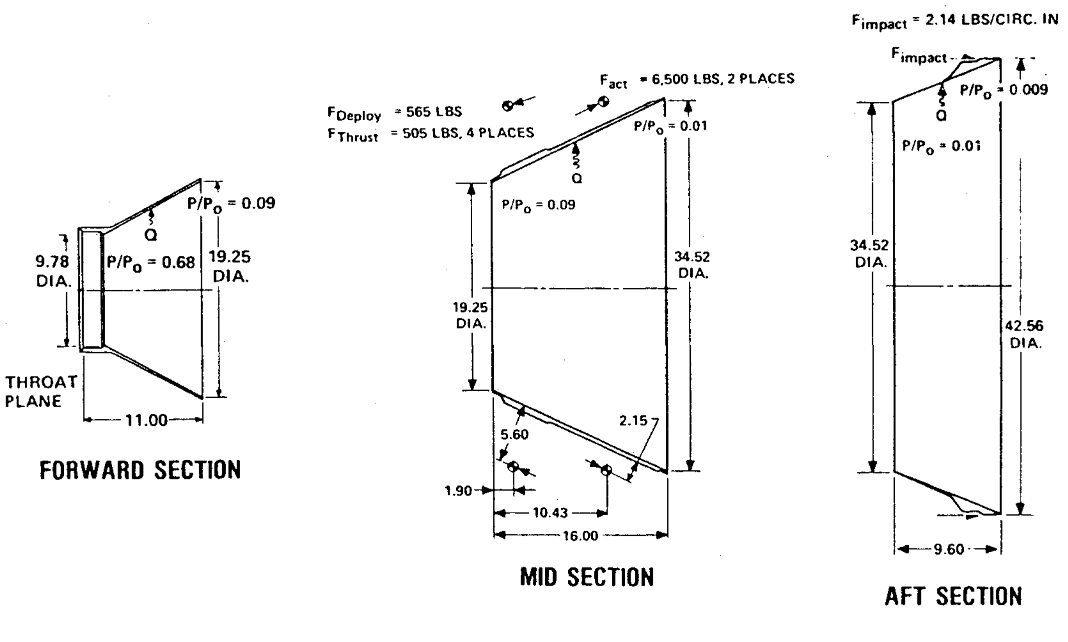


Fig. 2 Exit cone sectors with dimensions and loading conditions indicated.

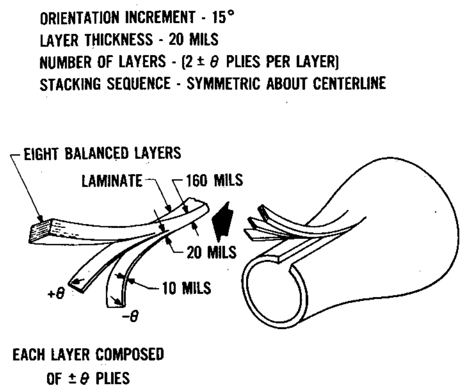
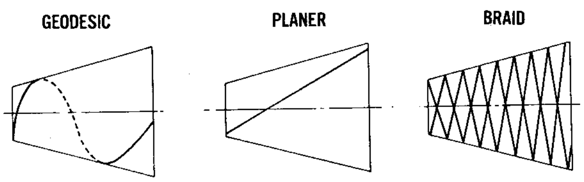


Fig. 3 Preselected design parameters.



ANGLE AT START OF SECTOR 1	SECTOR 2	SECTOR 3	FABRICATED BY
0	0	0	GORE STRIPS
15	15	0	PLANAR WRAP
	0	0	GEODESIC
30	30	30	PLANER WRAP
	15	0	GEODESIC
	75	90	BRAID
45	15	0	GEODESIC
	75	90	BRAID
60	15	0	GEODESIC
	75	90	BRAID
75	15	15	GEODESIC
	90	90	BRAID
90	15	15	GEODESIC
	90	90	HOOP WRAP

Fig. 4 Fabrication constraints.

realizations of a 7^{b-c} fractional factorial experimental design where $b=4$ and $c=3$ is constructed. This defines 28 specific candidate designs to be evaluated for each sector. Experimental design is an efficient method for selecting experimental stimuli to be employed in determining the causes of variability in experimental response. Its use here allows the construction of an efficient sampling experiment for estimating the structural design response surface. The data obtained by performing the field analysis for each of 28 candidate designs for each sector are included in a stepwise multiple regression analysis[8] to construct a second-order polynomial function that approximates the actual laminate response function. The sample size of 28 is selected to provide a sufficient confidence level for the approximating function.

In implementation, the field analysis results for each sector are used to estimate separate response functions for longitudinal, transverse, and shear factors of safety in the principal material directions as well as for bending compliance. Based upon the computed response behavior, two trial second-order functions are constructed. These functions have the forms

$$\hat{y}^1 = b_0^1 + \sum_{j=1}^{4} b_j^1 \cos\theta_j + \sum_{i=1}^{4}\sum_{j=1}^{4} b_{ij}^1 \cos\theta_i \cos\theta_j$$

$$\hat{y}^2 = b_0^2 + \sum_{j=1}^{4} b_j^2 X_j + \sum_{i=1}^{4}\sum_{j=1}^{4} b_{ij}^2 X_i X_j \tag{1}$$

where $\hat{y}^1$ and $\hat{y}^2$ are estimates of the true response, θ_j the fiber orientation angle in laminae j, $X_i = \theta_i/15$, and b_j^k and b_{ij}^k functional coefficients determined by the regression analysis.

For each model, two descriptors of model validity are computed. The coefficient of correlation R^2 is a measure of the percentage of the variation in the actual response function that is explained by the regression model and the F ratio is an indication of the statistical significance of the constructed model. The F test is to select between the hypotheses

Null: Ho = regression model is not statistically different from zero
Alternate: Ha = regression model is significant

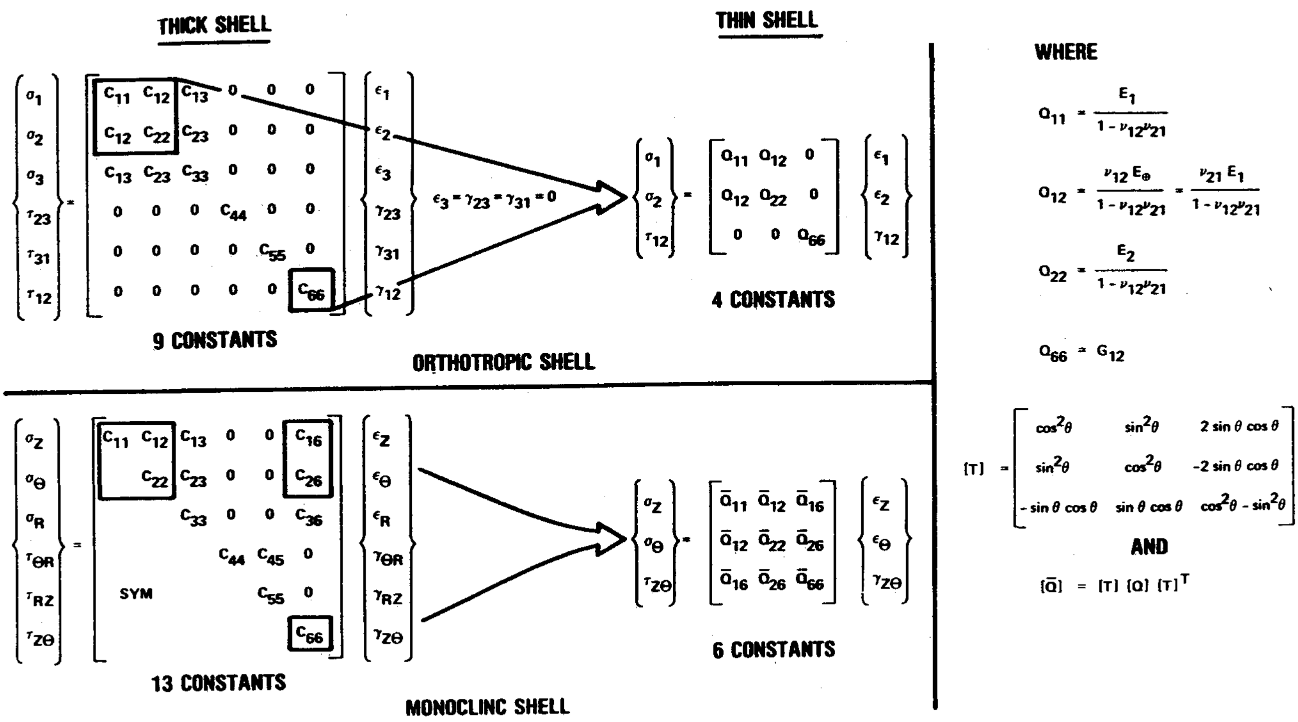


Fig. 5 Constitutive laws for the thin-shell approximation.

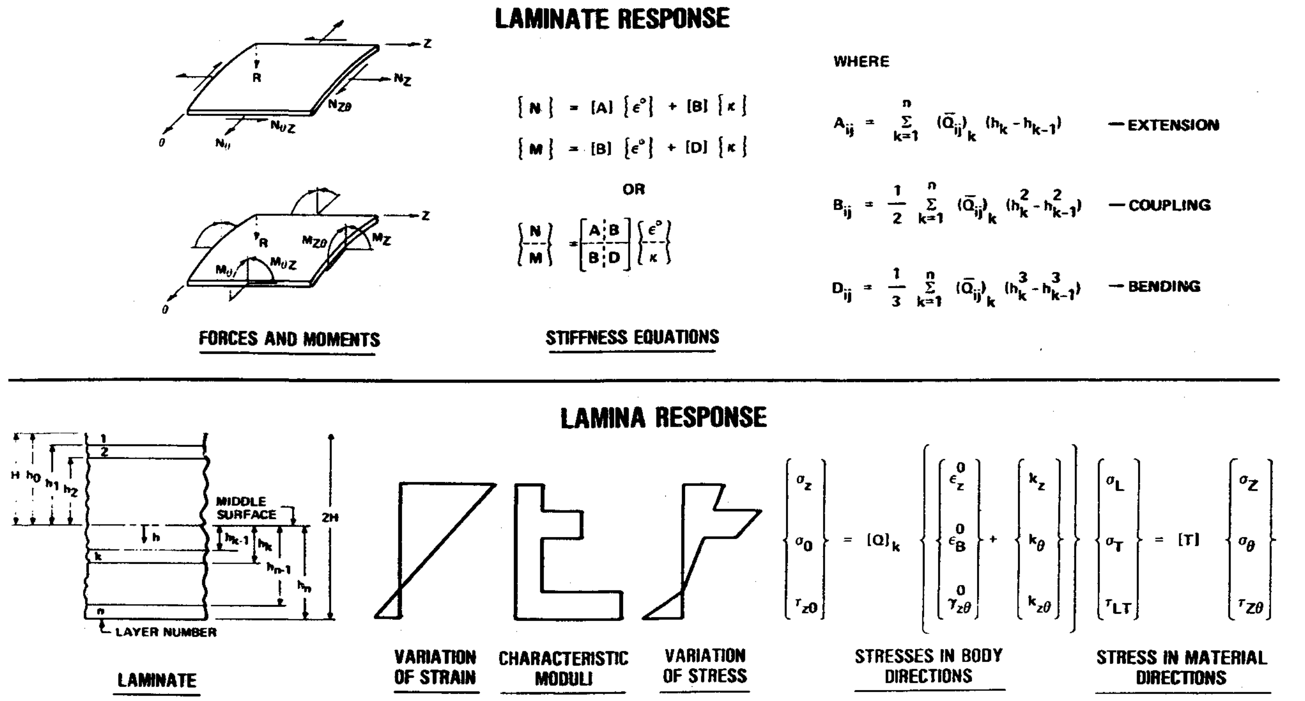


Fig. 6 Laminate behavior.

If the calculated value of the F ratio exceeds the corresponding tabulated value for the F distribution, then Ha is believed true. Otherwise, Ho is believed true.

The results obtained for each of the regression models are displayed in Tables 1 and 2. Examination of the tables shows that 3 of the 20 regression models are not statistically significant. This is a consequence of the difficulty of estimating a highly nonlinear and nonconvex response function using a polynomial of only second order. The obtained significant regression models are sufficient to define a conservative overall estimated response function that utilizes all of the statistically significant information contained in the regression models.

The estimated response function is conservative with respect to the structural design problem in that it is constructed by selecting the minimum of the estimated responses. In each sector of the cone, $\hat{y}_\ell^k$ represents the estimated response using function k for the factor of safety in fiber direction ℓ (longitudinal, transverse, and shear). Then,

$$\hat{y}^k_{\cdot} = \min_{\ell}\{\hat{y}^k_\ell\} \tag{2}$$

defines the minimum factor of safety across fiber direction and

$$Z = \min_{k}\{\hat{y}^k_{\cdot}\} \tag{3}$$

defines the overall minimum estimated factor of safety. The values of Z are taken to be the conservative final estimated laminate response function. The resulting tabulated design response estimates, Z, constitute the performance measure used in the optimization analysis.

Optimization

The optimization model is a dynamic program.[9] Structural synthesis is accomplished within the dynamic program through sector-by-sector design variable selection subject to variable choices in other sectors. The optimization model decision variables x_j^k are the angles of fiber orientation in laminae j of sector k. The state variables u_j^k are defined as the angles of fiber orientation in the adjacent more forward sectors. This

Table 1 Statistics for cosine-based polynomial regression models

Response function	R^2	$d.f$	F	F tabulated at 95% confidence	Hypothesis
Sector 1					
Longitudinal	0.560	10,25	3.181	2.24	Ha
Transverse	0.949	11,7	11.778	3.60	Ha
Shear	0.736	14,7	3.385	2.34	Ha
Sector 2					
Longitudinal	0.893	14,17	10.131	2.34	Ha
Transverse	0.935	14,15	15.274	2.45	Ha
Shear	0.586	14,17	1.721	2.34	Ho
Compliance	0.986	14,17	87.403	2.34	Ha
Sector 3					
Longitudinal	0.625	11,16	2.426	2.46	Ho
Transverse	0.746	12,14	3.426	2.53	Ha
Shear	0.755	12,14	3.321	2.50	Ha

Table 2 Statistics for surrogate variable-based polynomial regression models

Response function	R^2	$d.f$	F	F tabulated at 95% confidence	Hypothesis
Sector 1					
Longitudinal	0.557	11,24	2.744	2.21	Ha
Transverse	0.975	13,5	15.077	4.66	Ha
Shear	0.713	10,21	5.206	2.33	Ha
Sector 2					
Longitudinal	0.830	13,18	6.770	2.32	Ha
Transverse	0.919	14,15	12.219	2.45	Ha
Shear	0.673	12,19	3.264	2.31	Ha
Compliance	0.987	14,17	93.265	2.34	Ha
Sector 3					
Longitudinal	0.581	14,13	1.288	2.55	Ho
Transverse	0.775	13,13	3.440	2.57	Ha
Shear	0.791	14,13	3.503	2.55	Ha

state variable definition is employed in order to provide a means of imposing fabrication constraints upon the selection of decision variable values. Specifically, the state variables, along with restrictions on the extent to which the fiber orientation angles may change between sectors, defines the set of feasible designs for each sector.

The variables and functions that define the dynamic program are

x^k = the decision variables—the vector of fiber orientation angles for sector k
u^{k-1} = the state variables—for sector k, the current best choice of fiber orientation angles in sector $k-1$
$u^k = t_k(u^{k-1}, x^k)$ = the state variable transformation function that defines u^k in terms of u^{k-1} and x^k
$f_k(u^{k-1})$ = the overall optimal structural response function for sectors k-N given input state vector u^{k-1}
$g_k(x^k)$ = any applicable constraint function limiting the choice of a design vector x^k in sector k

Using these definitions, the structure of the dynamic program is illustrated in Fig. 7. As indicated in the figure, return and constraint functions may be associated with each sector, a decision vector is selected in each sector, and the choice of a decision vector causes a transformation of an input state vector to an output state vector.

Algebraically, the problem is to solve the set of dynamic programming recursion equations as follows.

$$f_3(u^2) = \underset{x^3}{\mathrm{opt}}\{r^3(u^2, x^3)\}$$
$$\text{subject to: } g_3(x^3) \geq 0$$

$$f_2(u^1) = \underset{x^2}{\mathrm{opt}}\{r^2(u^1,x^2)\Delta f_3\,[t_2(u^1,x^2)]\}$$
$$\text{subject to: } g_2(x^2) \geq 0$$

$$f_1(u^0) = \underset{x^1}{\mathrm{opt}}\{r_1(u^0,x^1)\Delta f_2\,[t_1(u^0,x^1)]\}$$
$$\text{subject to: } g_1(xY) \geq 0 \tag{4}$$

This set of recursion equations constitutes the dynamic programming optimization model. The equations are solved in reverse order [e.g., $f_3(u^2)$ first] in a stepwise fashion. First, an optimal choice of x^3 is identified for all possible input state vectors u^2. This is the first application of the "dynamic programming principle of optimality," which states that, given any history, an optimal policy has the property that it includes optimal decision variable choices forward from the point at which the history is fixed. Thus, for the exit cone design problem, any choice of variables that yields a particular value of

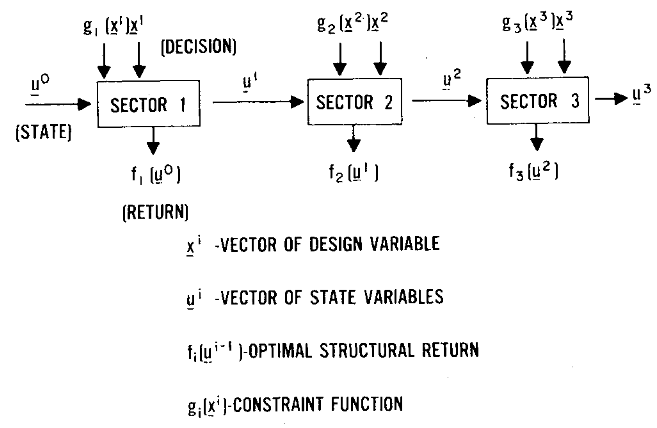


Fig. 7 Generic structure of three-stage dynamic program.

Table 3 Optimal solutions

Sector	Architecture	Safety factor estimated	Safety factor computed
1	(90/0/90/75)		
2	$(90/0/90_2)_s$	2.08	1.1
3	$(90/0/90_2)_s$		
1	$(90/15/90/75)_s$		
2	$(90/0/90_2)_s$	2.0	1.1
3	$(90/0/90_2)_s$		
1	$(30/90/0/90)_s$		
2	$(30/90/0/90)_s$	1.7	1.2
3	$(30/90/0/90)_s$		
1	$(30/90/0/90)_s$		
2	$(15/90/0/90)_s$	1.7	1.5
3	$(0/90/0/90)_s$		
1	$(60/90/0/75)_s$		
2	$(15/90/0/90)_s$	1.71	1.2
3	$(0/90/0/90)_s$		

the state vector u^2 will be part of an optimal solution only if the subsequent choice of x^3 is optimal with respect to that particular value of u^2.

Within the nesting framework implied by the state variable transformation equations, a second application of the principle of optimality mandates an optimal choice of decision vectors in both sectors 2 and 3 given a state vector u^1. Similar arguments apply for the initial state of vector u^0. All possible values of the initial state vector are considered in the present analysis and a search for the global optimum over all initial states is used to identify the optimal design.

An important feature of the employed form of the generic recursion equations defined above is that the composed return functions are maximized, while the composition operator (denoted by Δ) is the selection of the minimum of the quantities composed. Thus, the general form of the recursion equation is

$$f_k(u^{k-1}) = \max_{x^k}\left\{\min\left(r_k(u^{k-1}, x^k), f_{k+1}\,[t_k(u^{k-1}, x^k)]\right)\right\} \tag{5}$$

for which the optimum is the decision vector x^k that maximizes the minimum factor of safety over sectors k-N.

This form of the recursion is particularly appropriate for the laminated conical shell because it represents the strength of a chain of sectors by the strength of the weakest sector. Maximizing the strength of the weakest sector implies the choice of an acceptable overall design.

Recalling that $Z_k(x^k)$ is the estimated structural response associated with the selection of the vector of fiber orientation angles x^k in sector k the implemented form of the dynamic program is

$$f_3(u^2) = \max_{x^3 \epsilon h(u^2)} [Z_3(x^3)]$$

$$f_2(u^1) = \max_{x^3 \epsilon h(u^1)} \{\min Z_2(x^2), f_3(x^2)]\}$$
$$\text{subject to: } g_2(x^2) - 200{,}000 \geq 0$$

$$f_1(u^0) = \max_{x^1 \epsilon h(u^0)} \{\min[Z_1(x^1), f_2(x^1)]\}$$
$$= \max_{u^0} f_1(u^0) \tag{6}$$

where the restriction $x^2 \epsilon h(u^{k-1})$ implies that the maximization be taken over only those vectors x^k that are feasible in terms of fabrication given an input vector u^{k-1} and 200,000 lb/in. is

Table 4 Summary of results of integrated shell analysis

Loading condition	Lamination	Safety factor	Failure mode	Critical ply, deg
Ignition	$(30/90/0/90)_s$	1.80	Shear	30
(Pressure only)	$(15/90/0/90)_s$	2.46	Longitudinal	90
	$(0/90/0/90)_s$	31.51	Longitudinal	90
Exit cone deployment	$(30/90/0/90)_s$	1.72	Shear	30
(Pressure only)	$(15/90/0/90)_s$	2.42	Longitudinal	90
	$(0/90/0/90)_s$	30.53	Longitudinal	90
Maximum pressure	$(30/90/0/90)_s$	1.86	Shear	30
(30 s in-depth heating)	$(15/90/0/90)_s$	1.95	Longitudinal	90
	$(0/90/0/90)_s$	1.80	Longitudinal	0
TVC load at	$(30/90/0/90)_s$	1.69	Shear	30
30 s conditions	$(15/90/0/90)_s$	2.16	Longitudinal	90
	$(0/90/0/90)_s$	1.81	Longitudinal	0
TVC and ENEC loads	$(30/90/0/90)_s$	1.71	Shear	30
at 30 s conditions	$(15/90/0/90)_s$	1.90	Longitudinal	90
	$(0/90/0/90)_s$	1.83	Longitudinal	0
Graphitization (4000°F)	$(30/90/0/90)_s$	1.64	Shear	30
Process stress	$(15/90/0/90)_s$	15.20	Longitudinal	90
	$(0/90/0/90)_s$	1.50	Longitudinal	0

the asymmetric bending compliance minimum. Also, note that it is only in the second sector that there is any further constraint upon the choice of x^k. The constraint in sector 2 requires that the bending load compliance criterion imposed by the thrust vector control actuator be met.

The five best solutions obtained are listed in Table 3. The feasibility of these designs are verified by application of the field analysis codes. On this basis, the fourth of the five solutions is taken as the selected design concept.

It should be noted that the field theory models treat each sector as a separate structure with boundary conditions only approximately representative of the displacement continuity between sectors. As a further verification of the selected design, an integrated shell model is analyzed using the field analysis codes. The results of this analysis for a variety of loading conditions are summarized in Table 4.

Sensitivity

It is appropriate to examine model sensitivity to the input model parameters. One enlightening approach to examining model sensitivity is to study the behavior of the computed solutions as the model constraints are relaxed or tightened. Bending compliance is an important model constraint. Tightening this constraint to 250,000 lb/in. reduces the number of feasible design solutions modestly, but does not change the identities of the optima. This suggests that the solutions found persist for changes in compliance requirements because they possess greater than minimal strength in the directions of the bending loads.

Relaxing the fabrication constraints significantly affects the model solutions. The fabrication constraints dictate the fiber orientations that may be applied in the same ply over more than one sector. They reflect the ability of the fibers to conform to various wrapping patterns. Relaxing these constraints implies that the fibers may be wrapped in more patterns. In this case, the constrained optima are still good solutions, but several solutions with higher minimum safety factors are found. This suggests that the fabrication constraints influence the solutions and that improvements in fabrication methods resulting in greater wrapping flexibility may lead to stronger designs. At the same time, it may be observed that the optimum estimated minimum factor of safety for the case in which the fabrication constraints are completely relaxed is only 20% greater than that for the constrained solution. Thus, the constrained solution is a relatively good design and the apparent opportunity for improvement using new fabrication techniques is not great.

Conclusions

The solutions obtained using the dynamic program form a family of similar designs that are reasonable when considered qualitatively. The frequent occurrence of hoop fibers (90 deg) assures resistance to internal pressure and thermal expansion, while the axial fibers (0 deg) provide stiffness in the direction of the applied mechanical loads. In addition, the absence of 45 deg fibers is reasonable in view of their propensity for shear failure. Thus, the analytical solutions obtained conform well to engineering judgments concerning the design problem.

An analytical approach to the design of a laminated conical shell has been demonstrated. For the imposed limitations to discrete fiber orientation angles, laminate symmetry, balance, and state-of-the-art fabrication methods, the technique efficiently locates the optimal fiber reinforcement orientation and stacking sequence. The method permits the decoupling of the field analysis codes and the optimization routine.

Several advantages of the approach are apparent. The procedure is completely general with respect to the type of field analysis code used. It is also general with respect to loading conditions and shell geometry. The optimization procedure employs a conservative response measure, thereby assuring that the selected design will meet the performance requirements. Finally, the stepwise approach to the analysis permits the designer complete visibility to and control of the procedure.

The optimal lamination pattern identified is to be validated by fabrication and testing. Results of material testing and analysis sensitivity studies will be combined to establish an envelope of maximum material properties for the application of interest. Extensions to the method in the form of an examination of the consequences of additional sampling in the construction of the response function, consideration of continuous decision variables, and the introduction of constraints governing wrapping tension and surface friction are planned. Such modifications to the methodology are promising areas of further research that may lead to even greater design flexibility.

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References

[1]Baldur, R. "Structural Optimization by Inscribed Hyperspheres," *ASCE Journal of Engineering Mechanics,* Vol. 98, June 1972.

[2]Markham, L. R. and Herakovich, C. T., "Optimum Design of Composite Laminates with Thermal Effects," NASA VPI-E 1976-11, June 1976.

[3]Venkayya, V. B., "Survey of Optimization Techniques in Structural Design," AFFDL-TM FBR-78-43, 1978.

[4]Jones, R. M., *Mechanics of Composite Materials,* McGraw-Hill Book Co., New York, 1975.

[5]Bushnell, D., "Stress, Stability, and Vibration of Complex Branched Shells of Revolution: Analysis and User's Manual for BOSOR4," Lockheed Missiles and Space Co., Inc., Rept. AD748639, March 1972.

[6]Cross, J. G. and Jones, R. M., "Finite Element Stress Analysis of Axisymmetric and Plane Solids with Different Orthotropic, Temperature-Dependent Material Properties in Tension and Compression," SAMSO-TR-71-103, 1971.

[7]Montgomery, D. C., *Design and Analysis of Experiments,* John Wiley & Sons, New York, 1976.

[8]Draper, N. R. and Smith, H., *Applied Regression Analysis,* John Wiley & Sons, New York, 1966.

[9]Nemhauser, G. L., *Dynamic Programming,* John Wiley & Sons, New York, 1966.